

Enrollment No: \_\_\_\_\_

Exam Seat No: \_\_\_\_\_

# C.U.SHAH UNIVERSITY

## Winter Examination-2018

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT2

Semester : 1

Date : 28/11/2018

Branch: B. Tech (All)

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1**

**Attempt the following questions:**

**(14)**

- a) If  $y = \frac{1}{x}$  then  $y_n$  equal to  
(A)  $\frac{(-1)^n n!}{x^n}$  (B)  $\frac{(-1)^n n!}{x^{n+1}}$  (C)  $\frac{(-1)^{n-1} (n-1)!}{x^n}$  (D) None of these
- b) If  $y = (10)^{2x}$  then  $y_n$  equal to  
(A)  $2^n (10)^{2x} (\log 10)^n$  (B)  $2^n (10)^{2x} (\log 2)^n$  (C)  $2^n (10)^{2x} (\log 10)^{n+1}$   
(D) none of these
- c) The series  $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$  represent expansion of  
(A)  $\cot^{-1} x$  (B)  $\tan^{-1} x$  (C)  $\sin^{-1} x$  (D)  $\sin x$   
If  $y = \log(1+x)$ , then  $x$  equal to  
(A)  $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}$  (B)  $1-\frac{y^2}{2!}+\frac{y^4}{4!}-\dots$  (C)  $y+\frac{y^2}{2!}+\frac{y^3}{3!}+\frac{y^4}{4!}+\dots$   
(D) none of these
- e)  $\lim_{x \rightarrow \infty} \frac{x^n}{e^x} = \underline{\hspace{2cm}}$   
(A) 0 (B) 1 (C) 2 (D) none of these
- f)  $\lim_{x \rightarrow 0} \frac{5^x - 3^x}{x} = \underline{\hspace{2cm}}$   
(A) 2 (B)  $\log 2$  (C)  $\log 15$  (D)  $\log\left(\frac{5}{3}\right)$
- g) If  $P = r \tan \theta$ , then  $\frac{\partial P}{\partial r}$  is equal to  
(A)  $\sec^2 \theta$  (B)  $\tan \theta$  (C)  $\tan \theta + r \sec^2 \theta$  (D)  $\frac{1}{2} \tan \theta$



- h)** If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , then  $\frac{\partial r}{\partial x}$  is equal to  
 (A)  $\sec \theta$  (B)  $\sin \theta$  (C)  $\cos \theta$  (D)  $\cosec \theta$
- i)** If  $f(x, y) = 0$ , then  $\frac{dy}{dx}$  is equal to  
 (A)  $\frac{\partial f / \partial x}{\partial f / \partial y}$  (B)  $\frac{\partial f / \partial y}{\partial f / \partial x}$  (C)  $-\frac{\partial f / \partial y}{\partial f / \partial x}$  (D)  $-\frac{\partial f / \partial x}{\partial f / \partial y}$
- j)** If  $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)}$  is equal to  
 (A) 1 (B) -1 (C) zero (D) none of these
- k)** If  $y = \cos \theta + i \sin \theta$ , then the value of  $y + \frac{1}{y}$  is  
 (A)  $2 \cos \theta$  (B)  $2 \sin \theta$  (C)  $2 \cosec \theta$  (D)  $2 \tan \theta$
- l)** If  $x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$  then  $x_1 x_2 x_3 \dots \text{to } \infty$  is  
 (A) -3 (B) -2 (C) -1 (D) 0
- m)** An eigenvalue of a square matrix  $A$  is  $\lambda = 0$ . Then  
 (A)  $|A| \neq 0$  (B)  $A$  is symmetric (C)  $A$  is singular (D)  $A$  is skew-symmetric
- n)** The product of the eigenvalues of  $\begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$  is  
 (A) 2 (B) 4 (C) 6 (D) 0

**Attempt any four questions from Q-2 to Q-8**

**Q-2**

**Attempt all questions**

(14)

- a)** If  $y = \frac{x}{x^2 + a^2}$  then find  $\mathbf{y}_n$ . (5)
- b)** Prove that  $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$  (5)
- c)** If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$  then show that  $\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}$ . (4)

**Q-3**

**Attempt all questions**

(14)

- a)** If  $y = \sin(m \sin^{-1} x)$  then prove that  

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$$
- b)** Prove that  $\cos^{-1}[\tanh(\log x)] = \pi - 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right)$  (5)
- c)** Evaluate:  $\lim_{x \rightarrow 0} \left( \frac{a}{x} - \cot \frac{x}{a} \right)$  (4)

**Q-4**

**Attempt all questions**

(14)

- a)** Evaluate:  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$  (5)



**b)** If  $u = \frac{y^2}{x}$ ,  $v = \frac{x^2}{y}$ , evaluate  $J = \begin{pmatrix} x, y \\ u, v \end{pmatrix}$  and  $J' = \begin{pmatrix} u, v \\ x, y \end{pmatrix}$  and hence verify that  $JJ' = 1$ . (5)

**c)** Expand  $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$  in powers of  $(x-3)$ . (4)

**Q-5** **Attempt all questions** (14)

**a)** If  $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$ . (5)

**b)** Evaluate:  $\lim_{x \rightarrow \frac{\pi}{4}} (1 - \tan x) \sec 2x$  (5)

**c)** Find nth derivative of  $\tan^{-1} x$ . (4)

**Q-6** **Attempt all questions** (14)

**a)** Using the formula  $R = \frac{E}{I}$ , find the maximum error and percentage of error in R if  $I = 20$  with a possible error of 0.1 and  $E = 120$  with a possible error of 0.05 and  $R = 6$ . (5)

**b)** Prove that  $(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2(a^2 + b^2)^{\frac{m}{2n}} \cos\left(\frac{m}{n} \tan^{-1} \frac{b}{a}\right)$ . (5)

**c)** Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ . (4)

**Q-7** **Attempt all questions** (14)

**a)** Reduce the matrix  $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$  to the normal form and find its rank. (5)

**b)** Using De Moivre's theorem prove that (5)

$$(i) \cos 5\theta = 5 \cos \theta - 20 \cos^3 \theta + 16 \cos^5 \theta$$

$$(ii) \sin 5\theta = 5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta$$

**c)** If  $\tan(\alpha + i\beta) = x + iy$  then prove that  $x^2 + y^2 + 2x \cot 2\alpha = 1$ . (4)

**Q-8** **Attempt all questions** (14)

**a)** Investigate for what values of  $\lambda$  and  $\mu$  the equations (5)

$x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $x + 2y + \lambda z = \mu$ , have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

**b)** Find the continued product of all the values of  $\left(\frac{1}{2} + i \frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ . (5)

**c)** Find the inverse of  $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$  by Gauss-Jordan reduction method. (4)

